A band theory of helicon propagation

1968 J. Phys. A: Gen. Phys. 1 363

(http://iopscience.iop.org/0022-3689/1/3/309)

A band theory of helicon propagation

A. C. BAYNHAM and A. D. BOARDMAN†
Royal Radar Establishment, Malvern, Worcs.
MS. received 11*th December* 1967, *in revised form* 14*th February* 1968

Abstract. The theory of helicon propagation in a periodic structure has been formulated. It assumes the plasma to be cold and collisionless, and introduces a periodic modulation of electron density or of external magnetic field. The dispersion equation is shown to break up into bands of allowed and forbidden propagation. Numerical results are presented for the density modulated case, and it is seen that the band gap is proportional to magnetic field and that the band edges move to higher frequency as the magnetic field is increased. Some comments on the possibility of constructing a tunable filter, based on helicon propagation, are included.

1. Introduction

The helicon is a transverse circularly polarized plasma wave which propagates along an externally applied magnetic field at frequencies well below both the plasma and cyclotron frequencies. Since its discovery, this wave has created widespread interest because of its propagation characteristics. It is a slow, relatively loss-free wave, and exists throughout a frequency range which is readily accessible to the experimentalist.

In this paper we discuss helicon propagation in a periodic structure. Experience of the way in which other waves behave in a periodic medium leads us to expect that the helicon dispersion relationship will break up into frequency bands of allowed and forbidden propagation. The width of these bands can also be expected to be a function of the periodicity and the magnitude of the periodic modulation. Thus a periodic structure will allow departures from the parabolic helicon dispersion relationship, which are determined by the choice of periodicity.

The periodic structures envisaged may be produced by modulation of either the plasma density or the external magnetic field. These possibilities can both be discussed using the analysis in §§ 2 and 3. Section 4 contains numerical results for helicon propagation normal to the plane of a metal–semiconductor sandwich.

2. The helicon wave equation

The wave equation which will be derived in this section describes propagation in a uniform medium. The solutions of this equation will be used in the following section to generate solutions appropriate to a periodic medium.

The plasma is assumed to be cold and collisionless: a condition which can be more readily approached in gaseous plasmas than in solid-state plasmas.

The linearized equation of motion of an electron of mass m and charge q, in the presence of a uniform external magnetic field $\mathbf{B}_0$ is

$$m\frac{\partial \mathbf{v}}{\partial t} = q\mathbf{e} + q\mathbf{v}\times\mathbf{B}_0 \tag{2.1}$$

where $\mathbf{e}$ is the electric field producing $\mathbf{B}$, the first-order magnetic field perturbation, and $\mathbf{v}$ is the velocity of the particle. If all perturbations are assumed periodic in time t, with frequency ω, we can re-express (2.1) as

$$-i\omega\mathbf{j} = \frac{N_0 q^2}{m}\mathbf{e} + \frac{q}{m}\mathbf{j}\times\mathbf{B}_0 \tag{2.2}$$

where N_0 is the carrier density, and the current density $\mathbf{j}$ is defined as $N_0 q\mathbf{v}$.

† Permanent address: Physics Department, University of Salford, Salford, Lancs.

The curl of equation (2.2) gives

$$-i\omega \operatorname{curl} \mathbf{j} = \frac{N_0 q^2}{m} \operatorname{curl} \mathbf{e} + \frac{q\mathbf{B}_0}{m} \frac{\partial \mathbf{j}}{\partial z} \tag{2.3}$$

where it is assumed that $\mathbf{B}_0 = (0, 0, B_0)$ and div $\mathbf{j} = 0$ which, if we neglect the displacement current, follows from

$$\operatorname{curl} \mathbf{B} = \mu_0 \mathbf{j}. \tag{2.4}$$

Following Legendy (1964) and Klozenberg, McNamara and Thonemann (1965) we elect to eliminate $\mathbf{j}$ and $\mathbf{e}$ from equation (2.3) with the aid of equation (2.4) and

$$\operatorname{curl} \mathbf{e} = i\omega \mathbf{B} \tag{2.5}$$

thereby obtaining an equation for $\mathbf{B}$. We finally obtain

$$\operatorname{curl}\operatorname{curl} \mathbf{B} - iA\frac{\partial}{\partial z}(\operatorname{curl} \mathbf{B}) + D\mathbf{B} = 0 \tag{2.6}$$

where $A = \omega_c/\omega$ and $D = \omega_p^2/C^2$. $\omega_c = qB_0/m$ is the cyclotron frequency, $\omega_p^2 = N_0 q^2/\epsilon m$ is the plasma frequency, ϵ is the dielectric constant and C is the velocity of light.

Equation (2.6) is basically the same as the one derived by the authors quoted above, although the present derivation proceeds from a different starting point.

3. Bloch wave solutions

We consider propagation, parallel to the z axis, of a helicon of wave number k, and introduce a step function electron density or magnetic field variation of the form shown in figure 1. This problem is similar to the multi-layer problems of optics, and to the well-known Kronig–Penney model of electron propagation in a one-dimensional periodic potential.

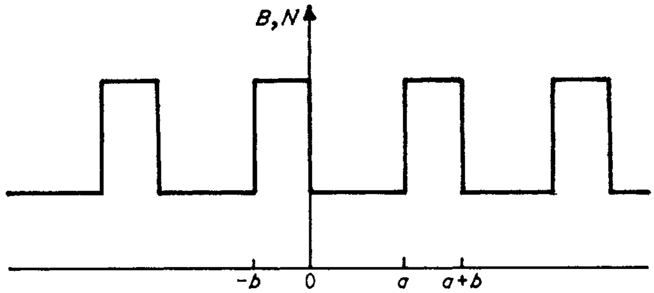

Figure 1. Periodic modulation of magnetic field B or particle density N.

We adopt the technique of Kronig and Penney (Dekker 1958). It is based upon Bloch's theorem (1928), which shows that, as a consequence of periodicity, solutions of the wave equation for a periodic medium are of the form

$$\mathbf{B} = \mathbf{f}_k(z)\, e^{ikz} \tag{3.1}$$

where k is the propagation constant and $\mathbf{f}_k(z)$ is a periodic function of z, having a period of $a+b$. This type of solution has also been used by Saunders and Baraff (1966) in a study of wave propagation along the interfaces of a multi-layer structure.

Substituting equation (3.1) into the uniform-medium wave equation (2.6), which it must also satisfy, we obtain equations for the components f_x and f_y of $\mathbf{f}(z)$ (the suffix k is now dropped for convenience). These can then be solved for region I extending over $0 < z < a$ and region II extending over $a < z < a+b$ of the unit cell of the system.

Equations (2.6) and (3.1), therefore, give

$$\begin{aligned}\hat{\mathbf{x}} &: f_x'' + 2ikf_x' - k^2 f_x + iA(k^2 f_y - 2ikf_y' - f_y'') - Df_x = 0 \\ \hat{\mathbf{y}} &: f_y'' + 2ikf_y' - k^2 f_y + iA(-k^2 f_x + 2ikf_x' + f_x'') - Df_y = 0\end{aligned} \tag{3.2}$$

where the prime denotes differentiation with respect to z. If we define $\psi = f_x + if_y$ (3.2) reduces to

$$\psi'' + 2ik\psi' + \left(\frac{D}{A-1} - k^2\right)\psi = 0. \tag{3.3}$$

Its solution is the circularly polarized helicon mode.

The solutions of (3.3) are

$$\psi = \mathrm{P}_{11}\exp\{i(\alpha - k)z\} + \mathrm{P}_{12}\exp\{-i(\alpha + k)z\}, \qquad 0 \leqslant z \leqslant a \tag{3.4}$$

$$\psi = \mathrm{P}_{21}\exp\{i(\beta - k)z\} + \mathrm{P}_{22}\exp\{-i(\beta + k)z\}, \qquad a \leqslant z \leqslant a+b \tag{3.5}$$

where

$$\alpha^2 = \frac{D_{\mathrm{I}}}{A_{\mathrm{I}} - 1} \quad \text{and} \quad \beta^2 = \frac{D_{\mathrm{II}}}{A_{\mathrm{II}} - 1}.$$

The interface regions are assumed to be sharp. In practice a density gradient or magnetic field gradient would exist over a finite region Δ. However, provided Δ is much less than the wavelength or the width of the unit cell it may safely be assumed that such a transition region can have little effect on the dispersion characteristics of the system. A discussion of this point can be found in Born and Wolf (1964) and Ginsberg (1964).

The boundary conditions at the interface between regions I and II of the unit cell are b_x, b_y, b_x' and b_y' continuous which means that ψ and ψ' are continuous.

These follow from standard electromagnetic theory, since the axial symmetry and semi-infinite transverse dimensions assumed here preclude the existence of surface currents at the interfaces.

The application of these boundary conditions together with the periodicity condition $\psi(-b) = \psi(a)$ gives a set of equations which reduce to

$$\frac{-(\beta^2 + \alpha^2)}{2\alpha\beta}\sin(\beta b)\sin(\alpha a) + \cos(\beta b)\cos(\alpha a) = \cos\{k(a+b)\} \tag{3.7}$$

Equation (3.7) is the dispersion relation describing helicon propagation, in a periodic structure.

4. Numerical results

The dispersion equation (3.7) has been solved numerically and the results are displayed in figure 2.

As has been pointed out earlier in the text, the theory is particularly applicable to a rare-gas plasma. However, in order to emphasize the appearance of frequency bands and the flexibility which is now available by a suitable choice of modulation, the parameters chosen actually describe a typical metal–semiconductor sandwich.

The figure shows that the band edges occur at $k = \pi/(a+b)$, and the propagation bands become progressively broader as the frequency is increased. Numerical calculations show that the band edges move to higher frequencies as the magnetic field is increased; for small changes of magnetic field the frequency axis is merely scaled by the fractional change.

These features suggest the possibility of constructing tunable filters based upon helicon propagation in periodic structures. Such filters are required to operate at high frequencies where a helicon-based filter would become increasingly selective and the transducing problems more tractable.

The foregoing remarks are orientated towards solid-state plasmas where the cold collisionless régime is difficult to realize in practice. The inclusion of collisional effects is not trivial. However, helicon propagation in infinite media is characterized by very small extinction coefficients. Therefore, since the periodicity is a design parameter it can

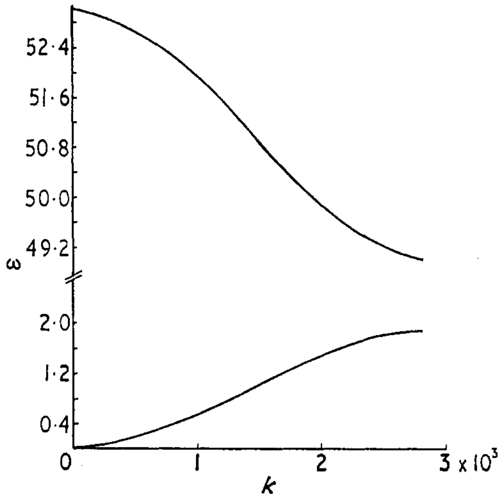

Figure 2. Frequency ω Hz plotted against reduced wave vector k for a magnetic field of 10 kG. Other parameters are $a = 10^{-3}$ cm, $b = 10^{-4}$ cm; $N = 10^{17}$ cm^{-3}, effective mass $= 0{\cdot}1\ m_0$ for $0 < z < a$; $N = 10^{23}$ cm^{-3}, effective mass $= 1\ m_0$ for $a < z < a+b$. The scale is changed, at the // mark, to $\omega \times 10^{-7}$ Hz.

readily be chosen to be less than the reciprocal extinction coefficient. Under these circumstances the wave amplitude is substantially unchanged between consecutive periodic boundaries, and the qualitative features of the theory developed in this paper are expected to remain unchanged in a calculation which includes scattering.

Acknowledgments

The authors are pleased to acknowledge the interest and helpful comments made by Dr. E. G. S. Paige.

References

Born, M., and Wolf, E., 1964, *Principles of Optics* (Oxford: Pergamon Press), p. 59.
Dekker, A. J., 1958, *Solid State Physics* (London: MacMillan) ,p. 243.
Bloch, F., 1928, *Z. Phys.*, **52,** 555.
Ginzburg, V. L., 1964, *The Propagation of Electromagnetic Waves in Plasmas* (Oxford: Pergamon Press), p. 192.
Klozenberg, J. P., McNamara, B., and Thonemann, P., 1965, *J. Fluid Mech.*, **21,** 545–63.
Legendy, C. R., 1964, *Phys. Rev.*, **135,** A1713–24.
Saunders, L. M., and Baraff, G. A., 1966, *J. Appl. Phys.*, **37,** 4551–6.